Image Denoising Model Based on Partial Differential Equation in Wavelet Transform Domain

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Abstract. Wavelet denoising model firstly decomposes image into high frequency and the low frequency information, and the noise characteristics are mainly concentrated in the high frequency information, but it often appears excessive image denoising phenomenon. Therefore, in order to improve the effect of image denoising, this paper proposes an image denoising model based on partial differential equations in wavelet transform domain. The partial differential equation is combined with the wavelet decomposition. According to the characteristics of the wavelet decomposition, the partial differential equation is processed in the high frequency information. The experimental results show that the model has a good effect on the peak signal to noise ratio after image denoising.

Introduction

In recent years, people have paid more and more attention to the problem of image denoising.At present, there are two main methods for image noise removal: one is pre-processing before imaging; the other is denoising processing after imaging[1].

Pre-processing requires high machine hardware, so more and more image denoising studies are placed on the post-imaging process.At present, there are three types of image denoising models[2]: the first is a space-based denoising model; the second is based on the wavelet transform domain denoising model[3]; the third is based on partial differential equations denoising model [4].Among them, the research on the latter two models is the most.

The reduced entropy of the wavelet transform after processing the image is characterized by removing noise according to different resolutions, de-correlation of the signal in the wavelet domain, selecting the basis function flexibly, and selecting the characteristics of the wavelet according to different filtering criteria, thereby making it get rid of noise, which does not depend on the size and shape of the algorithm window. The diffusion filtering of partial differential equations is anisotropic, so it can be diffused in different directions and different regions of the image, and the diffusion is related to the gradient value of the image, that is determined by the local structure of the image, so it can guarantee the correctness of smoothing and the protection of the edges. Where the image gradient value changes less, a strong filtering process is adopted to achieve a smooth noise suppression effect; where the image gradient value changes more, the diffusion effect is weakened, thus preserving local information and protecting edge features.

An image enhancement model based on wavelet transform and fractional differential is proposed in literature [5], which can improve the image denoising effect. However, there will be problems of excessive denoising, such as loss of detailed information of the image, blurring of the edges, and different denoising effects for different images. Literature [6] and [7] proposed an image denoising model based on partial differential equations, which can preserve the detailed information as much as possible while denoising.However, as the number of iterations increases, the gray level of the image tends to be constant, which produces a "block" effect,which affects the quality of the image.

In this paper, we first analyze the scale correlation of wavelet sub-bands, then analyze the partial

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differential equation model, construct a new diffusion function based on its functional relations, and propose a new partial differential equation of wavelet transform domain denoising model.The model uses a new diffusion function, which uses wavelet transform to separate the high-frequency and low-frequency information, and only nonlinearly spreads the high-frequency sub-bands. Therefore, not only the denoising effect but also the number of iterations can be reduced,which can avoid the "blocky" effect.The experimental results demonstrate the validity of the model.

Wavelet Scale Correlation and Threshold Analysis

Correlation after wavelet transform includes: intra-scale correlation, inter-scale correlation, intra-scale and inter-scale correlation[8].The third type of correlation includes not only the coefficients in the same sub-bands, but also the parent coefficient, so the correlation of such related information is the strongest of the three.The Gaussian noise image is decomposed by three-level wavelet, and the noise distribution is statistically analyzed. It is concluded that about 96% of the noise coefficients are distributed in the outermost two layers of sub-bands, that is the high frequency information.

Wavelet threshold filtering method is used for the smoothing, which can successfully separate the high and low frequency information of the image, which plays a crucial role in the subsequent operation of the image. However, at the same time, due to its limitations in the process of denoising, or the characteristics of the noise component, it is inevitable to affect its denoising effect.

Partial Differential Equation Model Analysis

In literature [6], the PM model of partial differential equations is proposed. The idea is to construct a “conduction coefficient” that can be changed with the local features of the image during the smoothing process.Its diffusion function is: $g(x) = k^2 / (x^2 + k^2)$,(where k is the threshold).Let I_0 be the initial grayscale image, and I be the grayscale image in the change, ∇I is the gradient of a certain pixel of the image, $\zeta = \frac{\nabla I}{|\nabla I|}$ is the unit vector in the gradient direction, η is the tangent vector with the vertical, then:

$$\begin{cases} \zeta = \dfrac{1}{\sqrt{I_x^2 + I_y^2}} \begin{pmatrix} I_x \\ I_y \end{pmatrix} \\ \eta = \dfrac{1}{\sqrt{I_x^2 + I_y^2}} \begin{pmatrix} -I_y \\ I_x \end{pmatrix} \end{cases} \tag{1}$$

Which is: $\zeta \cdot \eta = 0$

Let $I_{\zeta\zeta}$ denote the directional derivative along the gradient direction of the edge, and $I_{\eta\eta}$ denote the directional derivative along the tangential direction of the edge.Then you can get:

$$\begin{cases} I_{\zeta\zeta} = \dfrac{I_x^2 I_{xx} + 2 I_x I_y I_{xy} + I_y^2 I_{yy}}{I_x^2 + I_y^2} \\ I_{\eta\eta} = \dfrac{I_x^2 I_{yy} - 2 I_x I_y I_{xy} + I_y^2 I_{xx}}{I_x^2 + I_y^2} \end{cases} \tag{2}$$

Go a step further:

PM model

$$I_{\eta\eta} + I_{\zeta\zeta} = I_{xx} + I_{yy} \tag{3}$$

formula is as follows:

$$\frac{\partial I(x,y,t)}{\partial t}=div[g(|\nabla I|)\nabla I]=\frac{\partial}{\partial x}[g(|\nabla I|)I_x]+\frac{\partial}{\partial y}[g(|\nabla I|)I_y]$$

$$=\frac{\partial g(|\nabla I|)}{\partial(|\nabla I|)}\frac{\partial(|\nabla I|)}{\partial x}I_x+\frac{\partial g(|\nabla I|)}{\partial(|\nabla I|)}\frac{\partial(|\nabla I|)}{\partial y}I_y+g(|\nabla I|)(I_{xx}+I_{yy}) \quad (4)$$

Let $g'(|\nabla I|)=\frac{\partial g(|\nabla I|)}{\partial(|\nabla I|)}$, then Eq.4 can be changed to:

$$\frac{\partial I}{\partial t}=[g'(|\nabla I|)(|\nabla I|)+g(|\nabla I|)]I_{\zeta\zeta}+g(|\nabla I|)I_{\eta\eta} \quad (5)$$

It is brought into the Eq.5, which is the diffusion function given by the PM model. After finishing:

$$\frac{\partial I}{\partial t}=\frac{k^2}{k^2+|\nabla I|^2}\frac{k^2-|\nabla I|^2}{k^2+|\nabla I|^2}I_{\zeta\zeta}+\frac{k^2}{k^2+|\nabla I|^2}I_{\eta\eta} \quad (6)$$

Eq.6 is analyzed: when $k\neq|\nabla I|$, the model diffusion is performed along the gradient direction of the edge and the gradient direction perpendicular to the edge, which ensures the denoising effect; at the same time, when $k=|\nabla I|$, the coefficient before $I_{\zeta\zeta}$ is zero, the coefficient before $I_{\eta\eta}$ is not zero, which guarantees that it is not smooth along the gradient direction, thus preserving local detail information.It can be seen that the selection of value of k is very important. When the value is too big, the number of iterations increases, and the amount of calculation increases, resulting in "blocky"; when the value is too small, the denoising effect is not obvious.

Create a New Model

Constructing a Diffusion Function. According to the analysis of the literature [9] and the literature [10], the construction principle of $g(x)$ is that the diffusion is small in the place where the image gradient is large, and the diffusion is large in the place where the gradient is small, so that the model can filter out noise while terminating smoothas as far as possible in the edge area.The structure is as follows:

$$g(x)=\frac{1}{\ln(e+(x/k)^2)} \quad (7)$$

Proof is as follows:
According to Eq.5:

$$\frac{\partial I}{\partial t}=\left(\frac{1}{\ln(e+(x/k)^2)}\right)_x xI_{\zeta\zeta}+\frac{I_{\zeta\zeta}+I_{\eta\eta}}{\ln(e+(x/k)^2)} \quad (8)$$

Go a step further:

$$\frac{\partial I}{\partial t}=\left(\frac{1}{\ln(e+(x/k)^2)}-\frac{2x^2}{k^2\ln^2(e+(x/k)^2)(e+(x/k)^2)}\right)I_{\zeta\zeta}+\frac{1}{\ln(e+(x/k)^2)}I_{\eta\eta} \quad (9)$$

In order to protect the edge, the coefficient before $I_{\zeta\zeta}$ is zero.

Which means:

$$\frac{1}{\ln(e+(x/k)^2)}=\frac{2x^2}{k^2\ln^2(e+(x/k)^2)(e+(x/k)^2)} \quad (10)$$

It can be known from the Eq.10 that the two sides of the equation have the same definition domain, so the two sides of the equation are separately derived, and the result is obtained after the sorting:

$$k = \frac{1}{e} \cdot x = \frac{1}{e} \cdot |\nabla I| \quad (11)$$

It can be known from the Eq.9 that when $k \neq \frac{1}{e} \cdot |\nabla I|$, the coefficients before $I_{\zeta\zeta}$ and $I_{\eta\eta}$ are not zero. Thus, the effect of denoising is achieved; when $k = \frac{1}{e} \cdot |\nabla I|$, the coefficient before $I_{\zeta\zeta}$ is zero, thereby retaining local detail information.It can be seen from the Eq.11 that this model's value of k is smaller than the PM model's, so the amount of calculation is small and the number of iterations is also small.

Wavelet Decomposition. In the image denoising process based on wavelet sub-bands, only the high frequency sub-bands coefficients need to be considered. Because the different high frequency sub-bands have different directionalities, the direction template used in the nonlinear heterogeneous diffusion of high frequency sub-bands is: The horizontal template diffuses the high frequency sub-bands in the horizontal direction, the vertical template diffuses the high frequency sub-bands in the vertical direction, and the diagonal template diffuses the high frequency sub-bands in the diagonal direction.such as Table 1,Table 2,Table 3.

Table 1 Horizontal direction template

0	0	0
+	+	+
0	0	0

Table 2 Vertical direction template

0	+	0
0	+	0
0	+	0

Table 3 Diagonal direction template

+	0	+
0	+	0
+	0	+

Gradient Calculation. The four-neighbor structure is used to calculate the gradient. The formula is as follows:

$$|\nabla I|^2 = (I(i-1,j) - I(i,j))^2 + (I(i+1,j) - I(i,j))^2 + (I(i,j-1) - I(i,j))^2 + (I(i,j+1) - I(i,j))^2 \quad (12)$$

After discretizing the Eq.9, it is obtained:

$$I^{n+1} = I^n + \Delta t(\sum (g(|\nabla I|)\nabla I)) \quad (13)$$

Where Δt is the time step after discrete.

Algorithm Implementation Steps. ①I is expressed as the original noisy image; k takes a value of 1; Δt has an initial value of 0.1.

②A three-level wavelet transform is performed on I to determine the denoising of the outermost two sub-bands.

③The gradient value is calculated using Eq.12.

④In the outermost two sub-bands, Eq.13 is iteratively calculated in different direction templates.

⑤After the outermost sub-bands is denoised, the wavelet inverse conversion is performed.

⑥End.

Simulation Experiments and Analysis. Select three 256×256 test chart Lena, Peppers, Camera.The operating environment is Inter dual core, memory 2.0G, programming environment is VC6.0.The test pattern with 0.01 Gaussian noise is processed by the wavelet denoising model in literature [5], the PM model in literature [6], and the model in this paper.Take the Lena diagram as an example, the processing effect is shown in Figure.1.

(a)Lena image (b)Gaussian noise (c)Wavelet model (d)PM model (e)New model

Fig 1. Denoising results

It can be seen from Fig 1 that this new model has good denoising effect for Gaussian noise. Compared with the wavelet model and the PM model, the new model can preserve the local detail information while denoising, and because it considers the directionality of the noise, so the adaptive effect is stronger. Then, use the test chart Peppers and Camera to verify. As shown in Table 4 and Table 5, Gaussian noise with variance of 0.01-0.09 is added to the two graphs, and the PSNR value is obtained by comparing it with the wavelet threshold model and the PM model respectively.

Tab 4 Peppers test results

Gaussian noise	New model	Wavelet model	PM model
0.01	29.89	28.45	26.34
0.02	28.01	26.66	25.78
0.03	26.89	25.32	24.20
0.04	25.44	23.03	22.54
0.05	24.98	22.78	21.20
0.06	23.45	21.53	20.12
0.07	21.65	19.09	18.58
0.08	20.03	18.39	17.09
0.09	19.50	17.09	16.01

Tab 5 Camera test results

Gaussian noise	New model	Wavelet model	PM model
0.01	30.31	28.04	27.32
0.02	28.43	26.54	25.76
0.03	27.33	25.60	24.09
0.04	26.02	24.34	22.40
0.05	24.43	23.23	21.65
0.06	23.12	21.76	20.01
0.07	21.09	18.99	18.28
0.08	20.55	18.12	17.21
0.09	18.89	16.68	15.79

Conclusion

This paper constructs a new model that combines wavelet decomposition and partial differential equations.The model decomposes the image in the wavelet transform domain, then de-noises the partial differential equation in the high-frequency sub-bands, and finally reconstructs the wavelet.The model minimizes the detailed information of the image by diffusing only in the high frequency sub-bands, and reduces the amount of calculation by selecting a smaller threshold, thereby avoiding the "blocky" effect.The test results prove the validity of the model.

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